

# Statistical Orbit Determination



Lecture 12 – Real Measurements

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# Recap

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- Lecture 11 – Notes posted [here](#)
  - Real Measurements
    - Environment
    - One-Way Range System
- Questions
  - Post them to YouTube page



# Agenda

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- Environment
  - Atmosphere
  - Relativity
- Measurement systems
  - One way ranging
  - Two way ranging
  - Doppler measurements
  - Differenced measurements



# Environment – Atmosphere (1/2)

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- Troposphere

- Refractive to radio

- Troposphere delay, even though stratosphere 20%

- $\delta\rho = 10^{-6} \int N ds$ , where  $N$  is refractivity

- Dry refractive component contributes 90%

- $N_d = 77.6(P/T)$  where pressure is in mb

- $N_w = 3.73 \times 10^5 (e/T^2)$  where  $e$  is par. water vapor pressure

- Dispersive to optical

- Dependent on elevation



# Environment – Atmosphere (2/2)

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- Troposphere

- Dispersive to radio

- $\delta t = \alpha/f^2$ , where  $\alpha$  represents TEC

- $\alpha$  is positive for group delays (psuedorange) and negative for carrier phase

- Transmitting on two different  $f$  removes ionosphere for OD

- $\rho_c = \gamma_1\rho_1 - \gamma_2\rho_2 \mid \gamma_1 = \frac{f_1^2}{f_1^2 - f_2^2} \mid \gamma_2 = \frac{f_2^2}{f_1^2 - f_2^2}$

- No significant effect to optical

- Dependent on TEC

- Temporal and spatially dependent

- Starts at 80km and upward



# Environment – Relativity

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- General Relativity

- Satellite experience relativistic precession of perigee
  - $e = 0$ , clock beats faster or slower than gnd, dependent on altitude
- Clock varies w/ same orbital period for  $e \neq 0$ 
  - Still varies about mean frequency
  - Time offset correction,  $\Delta t_r = 2\bar{r} \cdot \bar{v}/c^2$
  - Time delay caused by Earth,  $\Delta t_{delay} = \frac{2\mu}{c^3} \ln \left[ \frac{r_1+r_2+\rho}{r_1+r_2-\rho} \right]$
  - Sagnac effect only concern for high precision clock transfer via sat



# Measurement Systems – One Way Range (1/2)

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- Global Positioning System (GPS)
  - GPS constellation, 6 orbital planes ( $i = 55^\circ$ ),  $T = 11\text{hr } 58\text{min}$
  - 3 Civil frequencies used
    - L1 and L2 transmit at 1575.42 and 1227.60 MHz, respectively
    - Transmit on carrier frequencies,  $154f_0$  and  $120f_0$ , where  $f_0 = 10.23$  MHz
    - Third frequency, L5 is  $115f_0$
  - Information superimposed on carrier
    - PRN is transmitted as bits on over carrier
    - Documented determination of bits attaches time to each
    - Cross-correlation with generated signal on ground with received signal
    - Codes transmitted are (C/A, 1023 bits, 1 ms), (P, 37 weeks, individual), (Y, AS)



# Measurement Systems – One Way Range (2/2)

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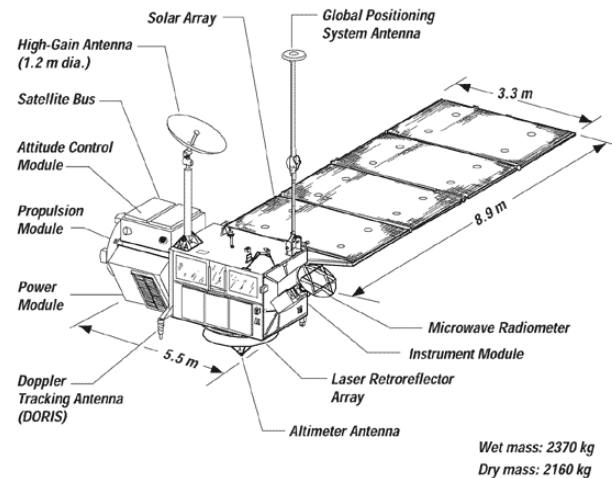
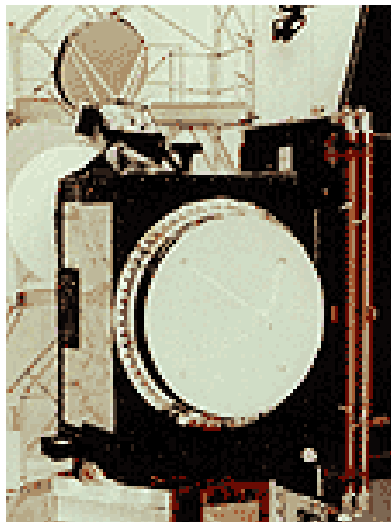
- Pseudorange measurement from L1 or L2
  - $\tilde{\rho} = \rho + c(\delta t_R - \delta t_T) + \delta\rho_{trop} + \delta\rho_{ion} + \epsilon$
  - True range requires true GPS position and receiver coordinates and true times
  - IN practice, may not be known with high accuracy
- Carrier phase gives more precise measurement
  - $\phi = \rho + c(\delta t_R - \delta t_T) + \lambda\tilde{N} + \delta\rho_{trop} - \delta\rho_{ion} + \epsilon$
  - $\phi$ , is measured phase range
  - $\tilde{N}$ , is integer phase ambiguity



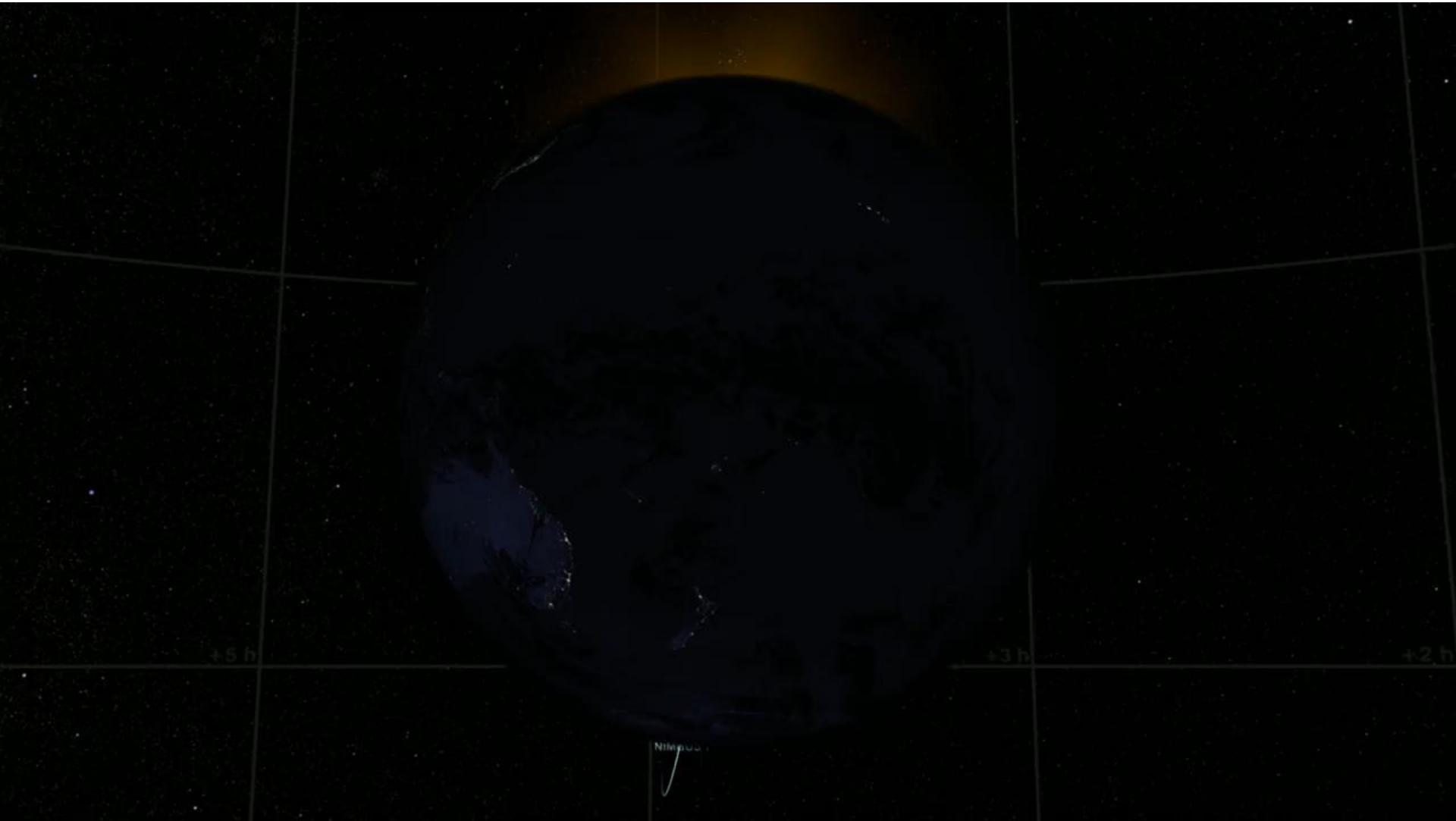


# Measurement Systems – Two Way Range (1/2)

- Satellite Laser Ranging (SLR)
  - Automated SLR-2000 system for autonomous concepts
  - TOPEX Poseidon passive reflector
    - 192 quartz corner-cubes mounted in two concentric rings around the altimeter antenna
    - Allows radial position estimate to within 3 cm
- TOPEX/Poseidon altimeter
  - TOPEX uses two frequencies to enable ionosphere correction (13.6 GHz/ 5.3 GHz)
  - Poseidon uses single frequency and other data to correct (13.65 GHz)



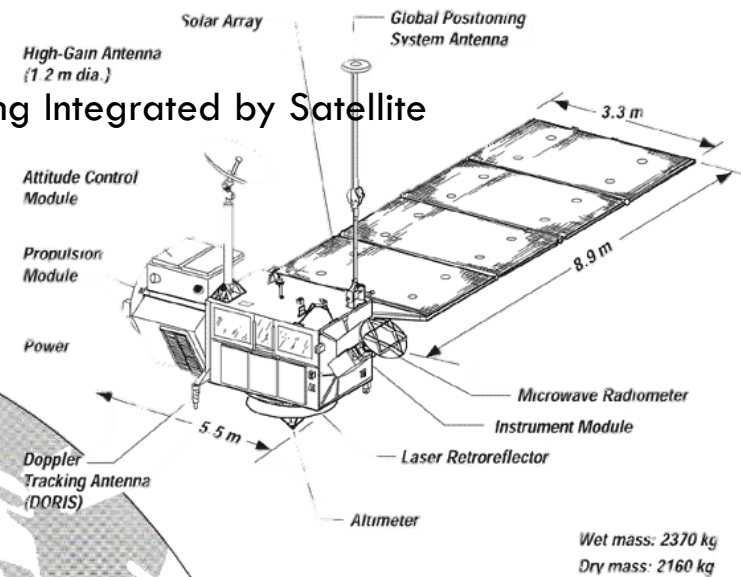
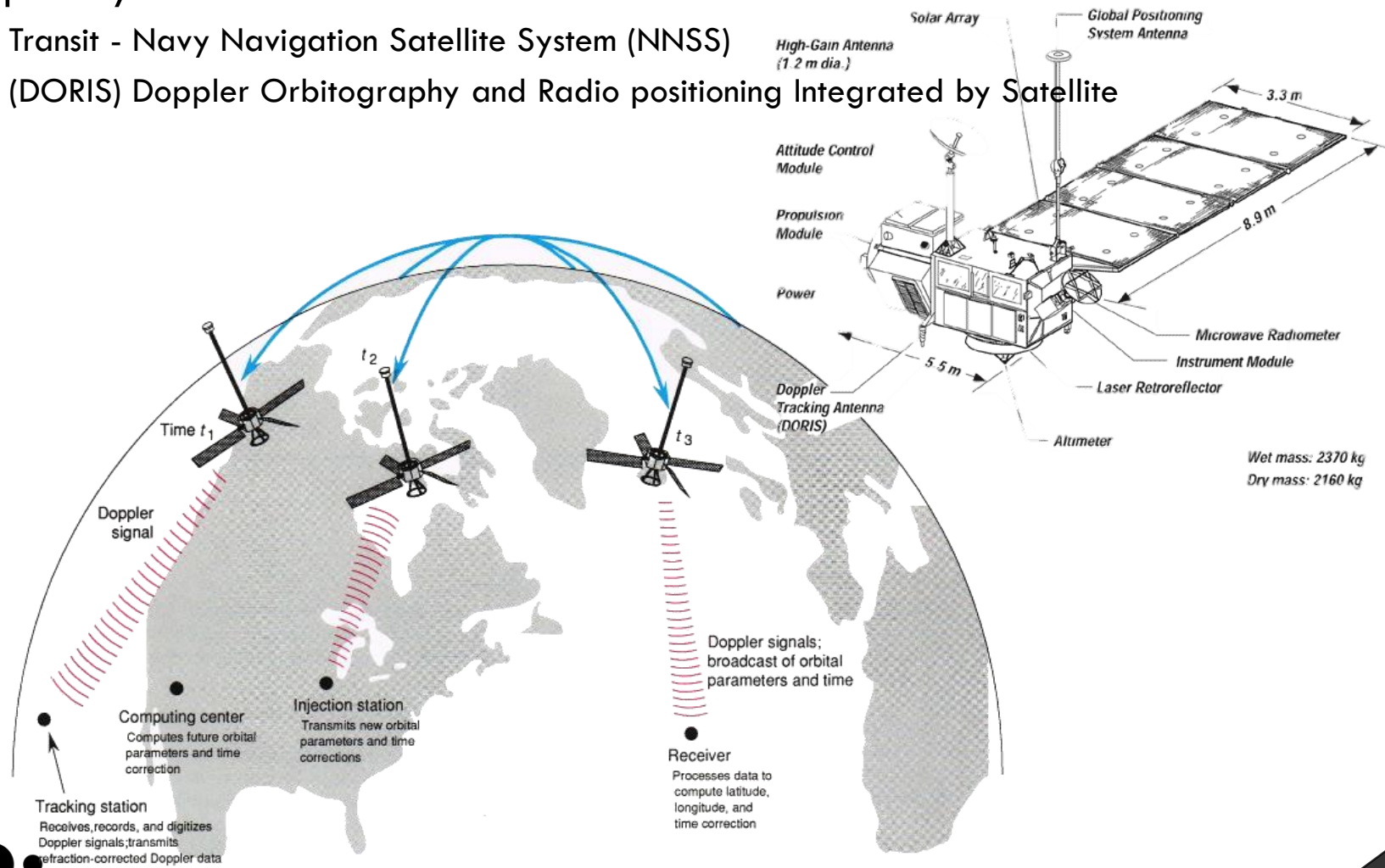
# Measurement Systems – Two Way Range (2/2)



# Measurement Systems – Doppler

- Doppler Systems

- Transit - Navy Navigation Satellite System (NNSS)
- (DORIS) Doppler Orbitography and Radio positioning Integrated by Satellite



# Measurement Systems – Differenced

- Differenced Measurements/Differencing
  - Measurements by a particular technique are differenced in special ways
  - Differencing removes or diminishes one or more error sources
  - Most common use
    - GPS pseudorange and carrier phase measurements
    - Altimeter measurements

